Additional Mathematics – Trigonometry

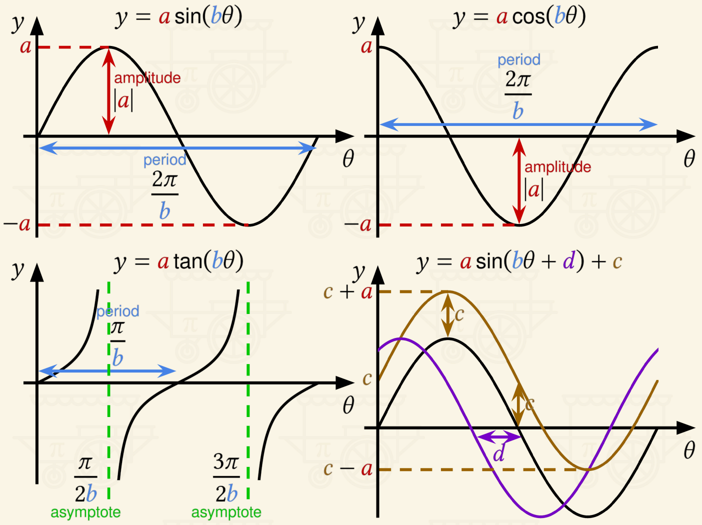
1. Foundations **2. Angles** **3. Identities**

1 Foundations

We have covered some basics in E-Math already, but let's dive deeper into the foundations of trigonometry.

Trigonometric Graphs

Here, we'll explore the graph of the tangent function. Unlike sine and cosine, it has a distinct shape with repeating vertical asymptotes due to its undefined values.



The graphs of $y = \sin(bx)$ and $y = \cos(bx)$ complete b cycles in $0 \leq x \leq 2\pi$.
The graph of $y = \tan(bx)$ complete b cycles in $0 \leq x \leq \pi$.

Example 1:

A curve has the equation $y = 5\sin(2\theta) - 3$.

a) State its amplitude, range, and period.

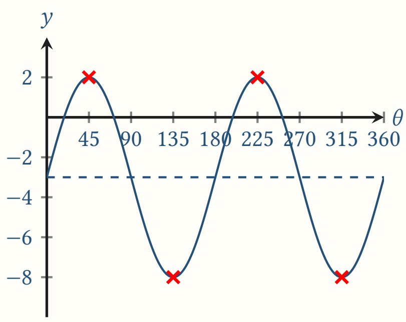
b) State the maximum and minimum points within $0^\circ \leq \theta \leq 360^\circ$.

c) Plot its graph for $0^\circ \leq \theta \leq 360^\circ$.

a) – The amplitude is 5.
– The range is $-8 \leq y \leq 2$.
– The period is $\frac{360^\circ}{2} = 180^\circ$.

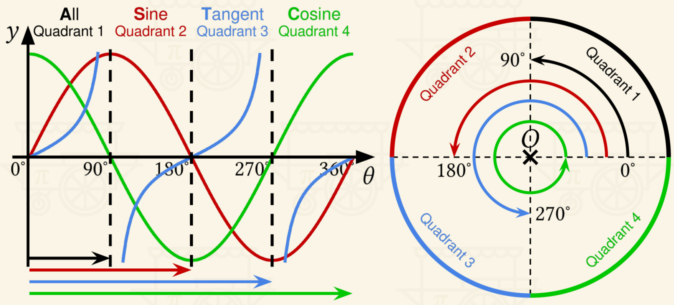
b) – The maximum points are $(45^\circ, 2)$ and $(225^\circ, 2)$.
– The minimum points are $(135^\circ, -8)$ and $(315^\circ, -8)$.

c)



Quadrants

The cyclic nature of trigonometric functions causes their signs to change across angles. Quadrants systematically track these changes, directly reflecting trigonometric ratios on the unit circle.



All	: Quadrant 1	$(0^\circ$ to $90^\circ)$
Sine	: Quadrant 2	$(90^\circ$ to $180^\circ)$
Tangent	: Quadrant 3	$(180^\circ$ to $270^\circ)$
Cosine	: Quadrant 4	$(270^\circ$ to $360^\circ)$

ASTC is a mnemonic that helps us remember which trigonometric functions are positive in each quadrant.

ASTC and basic angles (which we skip) help students manage the confusing nature of these graphs. However, they're mostly shortcuts for O-Levels that can sometimes cause more confusion. Understanding the unit circle and the graphs can lead to a deeper understanding.

Example 1:

a) Which quadrant does the angle 200° lie?

b) Which quadrant does the angle $-\frac{7\pi}{3}$ rad lie?

a) – 200° is in range $(180^\circ, 270^\circ)$
$\Rightarrow$ Angle lies in Quadrant 3.

b) – $-\frac{7\pi}{3} \times \frac{180^\circ}{\pi} = -420^\circ$ (convert to degrees)
– $-420^\circ + 2 \times 360^\circ = 300^\circ$ (translate to $0^\circ \leq \theta \leq 360^\circ$)
– 300° is in range $(270^\circ, 370^\circ)$
$\Rightarrow$ Angle lies in Quadrant 4.

Example 2:

If $\sin(\theta) = 0.866$ and $\cos(\theta) < 0$, find θ.

– $\sin(\theta) = 0.866$
$\theta = 60^\circ$ (Quadrant 1) or $\theta = 120^\circ$ (Quadrant 2)
– $\cos(\theta) < 0$ and θ is in Quadrant 2 or 3.
$\Rightarrow$ $\theta = 120^\circ$

Example 3:

Chloe is trying to determine where her cruise ship is heading. She observes the North Star and confirms that the ship is traveling somewhere northward. If she knows that $\sin(\theta) = 0.6$, where θ is the bearing of the ship (measured clockwise from the north), in which direction is the ship heading?

– $\sin(\theta) = 0.6$
$\theta = 36.9^\circ$ (north-east) or $\theta = 143^\circ$ (south-east)
– Ship is heading somewhere north.
$\Rightarrow$ The ship is traveling at a bearing of $\theta = 36.9^\circ$.

2 Angles

Let's meet the key angles you'll need again and again.

Reciprocal Trigonometric Functions

Reciprocal trigonometric functions gives 1 divided by the original ratio.

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Reciprocal Identity

- $\sin(\theta) \times \csc(\theta) = 1$

Some textbooks use cosec instead of csc to represent cosecant. csc is the standard notation, but follow what your school uses.

Special Angles

Special angles stand out as rare cases where trigonometric ratios simplify to neat, exact values instead of messy decimals.

θ	0°	30°	45°	60°	90°
$\sin(\theta)$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos(\theta)$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\frac{\sqrt{3}}{1}$	undefined
$\csc(\theta)$	undefined	$\frac{2}{\sqrt{1}}$	$\frac{2}{\sqrt{2}}$	$\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{4}}$
$\sec(\theta)$	$\frac{2}{\sqrt{4}}$	$\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{2}}$	$\frac{2}{\sqrt{1}}$	undefined
$\cot(\theta)$	undefined	$\frac{\sqrt{3}}{1}$	1	$\frac{1}{\sqrt{3}}$	0

Example 1:

Simplify the following expressions, leaving them in surd form.

a) $\sin(30^\circ) + \cos(60^\circ)$

b) $\tan^2(30^\circ) - \cos^2(60^\circ)$

c) $\sec\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right)$

d) $\frac{\tan(60^\circ) + \tan(45^\circ)}{1 - \tan(60^\circ)\tan(45^\circ)}$

e) $\frac{\sec(30^\circ)}{\csc(30^\circ)} + \cot(30^\circ)$

a) – $\sin(30^\circ) + \cos(60^\circ)$
$= \frac{1}{2} + \frac{1}{2}$
$= 1$

b) – $\tan^2(30^\circ) - \cos^2(60^\circ)$
$= \left(\frac{\sqrt{3}}{3}\right)^2 - \left(\frac{1}{2}\right)^2$
$= \frac{3}{9} - \frac{1}{4}$
$= \frac{1}{12}$

c) – $\sec\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right)$
$= \frac{2}{\sqrt{3}} + \left(\frac{\sqrt{3}}{2}\right)^2$
$= \frac{8\sqrt{3} + 3 \times 3}{12}$
$= \frac{9 + 8\sqrt{3}}{3}$

d) – $\frac{\tan(60^\circ) + \tan(45^\circ)}{1 - \tan(60^\circ)\tan(45^\circ)}$
$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3} \times 1)}$
$= \frac{(\sqrt{3} + 1)(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})}$
$= \frac{4 + 2\sqrt{3}}{-2}$
$= -2 - \sqrt{3}$

e) – $\frac{\sec(30^\circ)}{\csc(30^\circ)} + \cot(30^\circ)$
$= \frac{\frac{1}{\cos(30^\circ)}}{\frac{1}{\sin(30^\circ)}} + \cot(30^\circ)$
$= \tan(30^\circ) + \cot(30^\circ)$
$= \frac{1}{\sqrt{3}} + \sqrt{3}$
$= \frac{4\sqrt{3}}{3}$

Example 2:

a) Given that $\sin(\theta) = \frac{5}{13}$ where θ is an obtuse angle, find $\cot(\theta)$.

– $\sin(\theta) = \frac{\text{Opp}}{\text{Hyp}} = \frac{5}{13}$

– $\text{Adj} = \pm\sqrt{\text{Hyp}^2 - \text{Opp}^2}$
$= \pm\sqrt{13^2 - 5^2}$
$= \pm 12$

– $\tan(\theta) = \frac{\text{Opp}}{\text{Adj}} = \pm\frac{5}{12}$

– θ is obtuse.

$\Rightarrow$ θ lies in Quadrant 2.

$\Rightarrow$ tan is negative.

$\Rightarrow$ $\tan(\theta) = -\frac{5}{12}$

– $\cot(\theta) = \frac{1}{\tan(\theta)} = -\frac{12}{5}$

Inverse Trigonometric Functions

Inverse trigonometric functions find the angle from a given ratio.

$$\arcsin(\theta) = \sin^{-1}(\theta) \qquad \arccos(\theta) = \cos^{-1}(\theta) \qquad \arctan(\theta) = \tan^{-1}(\theta)$$

Inverse Identity

- $\sin[\sin^{-1}(\theta)] = \theta$

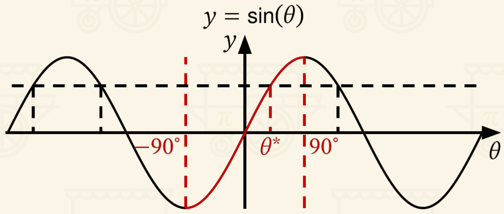
While "inverse" and "reciprocal" may seem interchangeable in ordinary algebra, they are not the same in trigonometry. The following is the reciprocal of sine, ***not*** the inverse function and they are not the same.

$$[\sin(\theta)]^{-1} = \frac{1}{\sin(\theta)} = \csc(\theta)$$

Principal Value

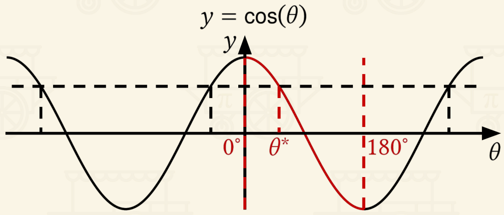
Because trigonometric functions repeat their values infinitely, using an inverse trigonometric function would give us infinitely many possible angles. To avoid this, we define a specific angle called the principal value. It is the unique solution within a fixed range, chosen so that each output corresponds to exactly one input, making the inverse function well-defined.

$y = \sin(\theta)$



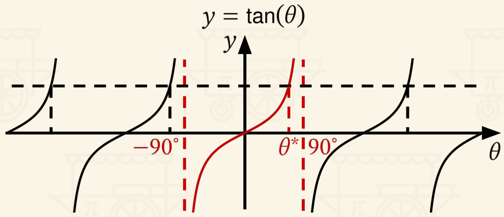
Principal Value (θ^*) : $-90^\circ \le \sin^{-1}(y) \le 90^\circ$

$y = \cos(\theta)$



Principal Value (θ^*) : $0^\circ \le \cos^{-1}(y) \le 180^\circ$

$y = \tan(\theta)$



Principal Value (θ^*) : $-90^\circ < \tan^{-1}(y) < 90^\circ$

Example 1:
Given that $2\sin(\theta) = \sqrt{3}$.

a) Find the principal value (θ^*) of θ, expressed in radians.

b) Find all values of θ within the range $-2\pi \le \theta \le 2\pi$.

a) – $2\sin(\theta) = \sqrt{3}$

$\sin(\theta) = \frac{\sqrt{3}}{2}$

– $\theta^* = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

b) – $\theta_1 = \pi - \theta^* = \frac{2\pi}{3}$ (dual solution)

– $(\theta_2, \theta_3) = (\theta^*, \theta_1) - 2\pi$ ($-2\pi \le \theta \le 0$ cycle)

$= \left(-\frac{5\pi}{3}, -\frac{4\pi}{3}\right)$

– Within $-2\pi \le \theta \le 2\pi$,

$\theta = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

3 Identities

Probably the most important part for exams.

Basic Identities

Trigonometric identities are equations that are always true for all angles. These basic identities help simplify expressions and solve equations, forming the foundation for many other formulae.

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\sec^2(\theta) = 1 + \tan^2(\theta)$
- $\csc^2(\theta) = 1 + \cot^2(\theta)$

Example 1:
Prove the identity $\frac{1}{\sec^2(\theta)} + \frac{1}{\csc^2(\theta)} = 1$.

– $\frac{1}{\sec^2(\theta)} + \frac{1}{\csc^2(\theta)} = \cos^2(\theta) + \sin^2(\theta) = 1$

Example 2:
Given that $\tan^2(\theta) = \frac{3}{5}$, find exactly $\sin^2(\theta)$.

– $\cot^2(\theta) = \frac{1}{\tan^2(\theta)} = \frac{5}{3}$

– $\csc^2(\theta) = 1 + \cot^2(\theta) = 1 + \frac{5}{3} = \frac{8}{3}$

– $\sin^2(\theta) = \frac{1}{\csc^2(\theta)} = \frac{3}{8}$

Example 3:
Given that $\sec(\theta) - \tan(\theta) = x$, prove that $\sec(\theta) + \tan(\theta) = \frac{1}{x}$.

– $\sec(\theta) + \tan(\theta)$

$= \frac{1}{\cos(\theta)} + \frac{\sin(\theta)}{\cos(\theta)}$

$= \frac{1 + \sin(\theta)}{\cos(\theta)} \times \frac{1 - \sin(\theta)}{1 - \sin(\theta)}$

$= \frac{1 - \sin^2(\theta)}{\cos(\theta)[1 - \sin(\theta)]}$

$= \frac{\cos^2(\theta)}{\cos(\theta)[1 - \sin(\theta)]}$

$= \frac{\cos(\theta)}{1 - \sin(\theta)}$

$= \left[\frac{1}{\cos(\theta)} - \frac{\sin(\theta)}{\cos(\theta)}\right]^{-1}$

$= [\sec(\theta) - \tan(\theta)]^{-1}$

$= \frac{1}{x}$

Addition Formulae

The addition formulae are useful when working with special angles. They also form the building blocks for advanced identities.

- $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
- $\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
- $\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$

Example 1:
Evaluate the following exactly.

a) $\sin(105^\circ)$ b) $\frac{1 + \tan(15^\circ)}{1 - \tan(15^\circ)}$

a) – $\sin(105^\circ)$

$= \cos(105^\circ - 90^\circ)$

$= \cos(60^\circ - 45^\circ)$

$= \cos(60^\circ)\cos(45^\circ) + \sin(60^\circ)\sin(45^\circ)$

$= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$

$= \frac{(1+\sqrt{3})\sqrt{2}}{2}$

b) – $\frac{1 + \tan(15^\circ)}{1 - \tan(15^\circ)}$

$= \frac{\tan(45^\circ) + \tan(15^\circ)}{1 - \tan(45^\circ)\tan(15^\circ)}$ ($\tan(45^\circ) = 1$)

$= \tan(45^\circ + 15^\circ)$

$= \tan(60^\circ)$

$= \sqrt{3}$

Example 2:
Given that $\sin(\theta) = \frac{3}{5}$ and $\cos(\phi) = \frac{5}{13}$, where θ and ϕ are principal values, find exactly the following expression.

a) $\sin(\theta + \phi)$ b) $\tan(\theta + \phi)$

– $\sin(\theta) = \frac{\text{Opp}_\theta}{\text{Hyp}_\theta} = \frac{3}{5}$

$\text{Adj}_\theta = \sqrt{5^2 - 3^2} = 4$

– $\cos(\phi) = \frac{\text{Adj}_\phi}{\text{Hyp}_\phi} = \frac{5}{13}$

$\text{Opp}_\phi = \sqrt{13^2 - 5^2} = 12$

a) – $\cos(\theta) = \frac{4}{5}$ $\sin(\phi) = \frac{12}{13}$

– $\sin(\theta + \phi)$

$= \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$

$= \left(\frac{3}{5}\frac{5}{13}\right) + \left(\frac{4}{5}\frac{12}{13}\right)$

$= \frac{63}{65}$

b) – $\tan(\theta) = \frac{3}{4}$ $\tan(\phi) = \frac{12}{5}$

– $\tan(\theta + \phi)$

$= \frac{\tan(\theta) + \tan(\phi)}{1 - \tan(\theta)\tan(\phi)}$

$= \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4}\frac{12}{5}}$

$= \frac{\frac{63}{20}}{-\frac{16}{20}}$

$= -\frac{63}{16}$

Example 3:
Prove that $\cos(\theta + \phi)\cos(\theta - \phi) = \cos^2(\theta) - \sin^2(\phi)$.

– $\cos(\theta + \phi)\cos(\theta - \phi)$

$= [\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)][\cos(\theta)\cos(\phi) + \sin(\theta)\sin(\phi)]$

$= [\cos(\theta)\cos(\phi)]^2 - [\sin(\theta)\sin(\phi)]^2$

$= \cos^2(\theta)\cos^2(\phi) - \sin^2(\theta)\sin^2(\phi)$

$= \cos^2(\theta)[1 - \sin^2(\phi)] - [1 - \cos^2(\theta)]\sin^2(\phi)$

$= \cos^2(\theta) - \cos^2(\theta)\sin^2(\phi) - \sin^2(\phi) + \cos^2(\theta)\sin^2(\phi)$

$= \cos^2(\theta) - \sin^2(\phi)$

Double Angle Formulae

The double-angle formulae are useful when simplifying expressions or solving equations that involve doubled angles.

- $\sin(2A) = 2\sin(A)\cos(A)$
- $\cos(2A) = \cos^2(A) - \sin^2(A)$
  $= 2\cos^2(A) - 1$
  $= 1 - 2\sin^2(A)$
- $\tan(2A) = \dfrac{2\tan(A)}{1-\tan^2(A)}$

Example 1:

Solve the equation $4\cos(2\theta) + 5\sin(\theta) - 2 = 0$ for $0 \le \theta \le 2\pi$.

- $4\cos(2\theta) + 5\sin(\theta) - 2 = 0$
  $4[1 - 2\sin^2(\theta)] + 5\sin(\theta) - 2 = 0$
  $8\sin^2(\theta) - 5\sin(\theta) - 2 = 0$
  $\sin(\theta) = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(8)(-2)}}{2(8)}$
  $= \dfrac{5 \pm \sqrt{89}}{16}$
  $\sin(\theta) = -0.277$ or $\sin(\theta) = 0.902$
- $\theta = \sin^{-1}(-0.277) = -0.281$ (Quadrant 4)
  $\theta = -0.281 + 2\pi = 6.01$ $(0 \le \theta \le 2\pi)$
  $\theta = 3\pi - (6.01) = 3.42$ (Quadrant 3)
- $\theta = \sin^{-1}(0.902) = 1.12$ (Quadrant 1)
  $\theta = \pi - 1.12 = 2.02$ (Quadrant 2)
- $\theta = 1.12, 2.02, 3.42, 6.01$

Example 2:

Prove the following identities.

a) $\dfrac{\sin(\theta)\cos(\theta)}{\cos^2(\theta) - \sin^2(\theta)} = \dfrac{1}{2}\tan(2\theta)$.b) $\dfrac{1-\tan^2(\theta)}{1+\tan^2(\theta)} = \cos(2\theta)$

c) $\tan(3\theta) = \dfrac{3\tan(\theta) - \tan^2(\theta)}{1 - 3\tan^2(\theta)}$

a) – $\dfrac{\sin(\theta)\cos(\theta)}{\cos^2(\theta) - \sin^2(\theta)}$
$= \dfrac{1}{2}\dfrac{2\sin(\theta)\cos(\theta)}{\cos^2(\theta) - \sin^2(\theta)}$
$= \dfrac{1}{2}\dfrac{\sin(2\theta)}{\cos(2\theta)}$
$= \dfrac{1}{2}\tan(2\theta)$

b) – $\dfrac{1-\tan^2(\theta)}{1+\tan^2(\theta)}$
$= \dfrac{1 - \dfrac{\sin^2(\theta)}{\cos^2(\theta)}}{1 + \dfrac{\sin^2(\theta)}{\cos^2(\theta)}}$
$= \dfrac{1 - \dfrac{\sin^2(\theta)}{\cos^2(\theta)}}{1 + \dfrac{\sin^2(\theta)}{\cos^2(\theta)}}$
$= \dfrac{\dfrac{\cos^2(\theta) - \sin^2(\theta)}{\cos^2(\theta)}}{\dfrac{\cos^2(\theta) + \sin^2(\theta)}{\cos^2(\theta)}}$
$= \dfrac{\cos^2(\theta) - \sin^2(\theta)}{\cos^2(\theta) + \sin^2(\theta)}$
$= \dfrac{\cos(2\theta)}{1}$
$= \cos(2\theta)$

c) – $\tan(3\theta)$
$= \tan(\theta + 2\theta)$
$= \dfrac{\tan(\theta) + \dfrac{2\tan(\theta)}{1-\tan^2(\theta)}}{1 - \tan(\theta)\dfrac{2\tan(\theta)}{1-\tan^2(\theta)}}$
$= \dfrac{\dfrac{3\tan(\theta) - \tan^3(\theta)}{1-\tan^2(\theta)}}{\dfrac{1-3\tan^2(\theta)}{1-tan^2(\theta)}}$
$= \dfrac{3\tan(\theta) - \tan^3(\theta)}{1 - 3\tan^2(\theta)}$

R-Formulae

The R-formulae is a technique to combine a sine term and a cosine term into a single sine (or cosine) term with a phase shift. This helps us simplify expressions, sketch graphs more easily, and find maximum or minimum values.

- $a\sin(\theta) \pm b\cos(\theta) = R\sin(\theta \pm \alpha)$
- $a\cos(\theta) \pm b\sin(\theta) = R\cos(\theta \mp \alpha)$

where $a, b > 0$, $R = \sqrt{a^2 + b^2}$, and $\alpha = \tan^{-1}\left(\dfrac{b}{a}\right)$.

Example 1:

Express $y = 7\cos(\theta) + 24\sin(\theta)$ in the form of $y = R\sin(\theta - \alpha)$, where $R > 0$. Hence, find the coordinates of the maximum and minimum points within $0 \le \theta \le 2\pi$.

- $R = \sqrt{7^2 + 24^2} = 25$
  $\alpha = \tan^{-1}\left(\dfrac{24}{7}\right) = 1.29$
  $y = 25\sin(\theta - 1.29)$
- Maximum occurs at $\sin(\theta - \alpha) = 1$.
  $y_{max} = R = 25$
  $\theta_{max} - \alpha = \sin^{-1}(1) = \dfrac{\pi}{2}$
  $\theta_{max} = \dfrac{\pi}{2} + 1.29 = 2.86$
  $(\theta_{max}, y_{max}) = (2.86, 25)$
- Minimum occurs at $\sin(\theta - \alpha) = -1$.
  $y_{min} = -R = -25$
  $\theta_{min} - \alpha = \sin^{-1}(-1) = -\dfrac{\pi}{2}$
  $\theta_{min} = -\dfrac{\pi}{2} + 1.29 = -0.284$
  $\theta_{min} = -0.284 + 2\pi = 6.00$ $(0 \le \theta \le 2\pi)$
  $(\theta_{min}, y_{min}) = (6.00, -25)$

Example 2:

Prove that $a\sin(\theta) + b\cos(\theta) = R\sin(\theta + \alpha)$, where $R = \sqrt{a^2 + b^2}$ is a valid choice.

Step 1): Expand the right-hand side.

- $R\sin(\theta + \alpha) = R(\sin(\theta)\cos(\alpha) + \cos(\theta)\sin(\alpha))$
  $= R\cos(\alpha)\sin(\theta) + R\sin(\alpha)\cos(\theta)$ — (1)
- $a\sin(\theta) + b\cos(\theta) = R\cos(\alpha)\sin(\theta) + R\sin(\alpha)\cos(\theta)$

Step 2): Equate coefficients.

- $a = R\cos(\alpha)$
  $\cos(\alpha) = \dfrac{a}{R}$ — (2)
- $b = R\sin(\alpha)$
  $\sin(\alpha) = \dfrac{b}{R}$ — (3)

Step 3): Continue from (1) using derived equations (2) and (3).

$R\sin(\theta + \alpha) = R\cos(\alpha)\sin(\theta) + R\sin(\alpha)\cos(\theta)$
$= R\left(\dfrac{a}{R}\right)\sin(\theta) + R\left(\dfrac{b}{R}\right)\cos(\theta)$
$= a\sin(\theta) + b\cos(\theta)$

Step 4): Show that $R = \sqrt{a^2 + b^2}$ is valid.

- $(2)^2 + (3)^2$:
  $\cos^2(\alpha) + \sin^2(\alpha) = \dfrac{a^2}{R^2} + \dfrac{b^2}{R^2}$
  $R^2[\cos^2(\alpha) + \sin^2(\alpha)] = a^2 + b^2$
  $R^2 = a^2 + b^2$
  $R = \sqrt{a^2 + b^2}$ is a valid choice.

Example 3:

A particle moves along a straight line such that its displacement at time t seconds is given by $x = 8\cos(2t) + 6\sin(2t)$.

a) Express x in the form $R\sin(2t + \alpha)$, where $R > 0$.

b) Determine the maximum displacement of the particle.

c) Find the first time at which the displacement is maximum.

a) – $R = \sqrt{8^2 + 6^2} = 10$

$\alpha = \tan^{-1}\left(\frac{6}{8}\right) = 0.644 \text{ rad}$

$x = 10\sin(2t + 0.644)$ units

b) – $x_{max} = R = 10$ units

c) – $\sin(2t + 0.644) = 1$

$2t + 0.644 = \frac{\pi}{2}$

$t = 0.464$ seconds

REACH US AT:

MATHHAWKER.COM.SG – ALL NOTES

@MATHHAWKER

@MATHHAWKER – LATEST UPDATES

PATREON.COM/MATHHAWKER – SUPPORT US :)