



Elementary Mathematics – Geometry part I

1. Triangle Trigonometry

2. Wave Trigonometry

3. Triangle Geometry

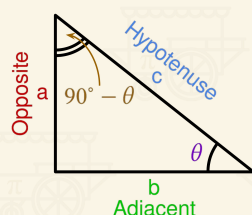
1

Triangle Trigonometry

Triangles are great! Trigonometry tells us how the sides and angles of our favourite 3-sided shape work together.

Right-angled Triangles

A summary of key identities and relationships in right-angled triangles.



Trigonometric Ratios

$$\begin{aligned} 1) \quad \sin(\theta) &= \frac{\text{Opp}}{\text{Hyp}} = \frac{a}{c} \\ 2) \quad \cos(\theta) &= \frac{\text{Adj}}{\text{Hyp}} = \frac{b}{c} \\ 3) \quad \tan(\theta) &= \frac{\text{Opp}}{\text{Adj}} = \frac{a}{b} = \frac{\sin(\theta)}{\cos(\theta)} \end{aligned}$$

Complementary Angles

$$\begin{aligned} 1) \quad \sin(\theta) &= \cos(90^\circ - \theta) \\ 2) \quad \cos(\theta) &= \sin(90^\circ - \theta) \\ 3) \quad \tan(\theta) &= \frac{1}{\tan(90^\circ - \theta)} \end{aligned}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

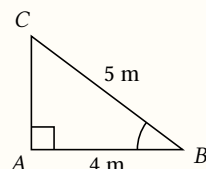
Opp Adj Hyp

“SOH CAH TOA” (or “TOA CAH SOH”) is a common mnemonic that helps students remember how sine, cosine, and tangent relate to the opposite, adjacent, and hypotenuse sides of a right-angled triangle.

Example 1:

In $\triangle ABC$, $BC = 5$ m, $AB = 4$ m, and $\angle CAB = 90^\circ$.

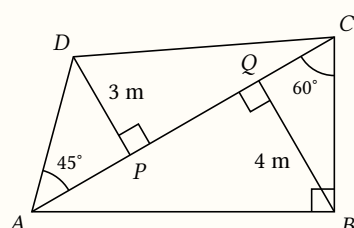
- a) Find length CA . b) Find $\angle ABC$.



$$\begin{aligned} \text{a) } & a^2 + b^2 = c^2 \\ & CA^2 + 4^2 = 5^2 \\ & CA = \pm\sqrt{9} = 3 \text{ m} \quad (\text{length can't be negative}) \\ \text{b) } & \sin(\theta) = \frac{\text{Opp}}{\text{Hyp}} \\ & \cos(\angle ABC) = \frac{4}{5} \\ & \angle ABC = \cos^{-1}\left(\frac{4}{5}\right) = 36.87^\circ \end{aligned}$$

Example 2:

In quadrilateral $ABCD$, APC is a straight line, $BQ = 4$ m, $PD = 3$ m, $\angle BCQ = 60^\circ$, $\angle DAP = 45^\circ$, and $\angle ABC = \angle BQA = \angle DPC = 90^\circ$. Find the total area of $ABCD$.



Step 1): Find area of $\triangle APD$.

$$\begin{aligned} & AP = 3 \text{ m} \quad (\text{isos. } \triangle) \\ & \text{Area}_{\triangle APD} = \frac{1}{2} \times \text{Base} \times \text{Height} \\ & = \frac{1}{2} \times 3 \times 3 \\ & = 4.5 \text{ m}^2 \end{aligned}$$

Step 2): Find area of $\triangle ABC$.

$$\begin{aligned} & \sin(60^\circ) = \frac{4}{BC} \\ & BC = 4.62 \text{ m} \\ & \tan(60^\circ) = \frac{AB}{4.62} \\ & AB = 8 \text{ m} \\ & \text{Area}_{\triangle ABC} = \frac{1}{2} \times 8 \times 4.62 = 18.48 \text{ m}^2 \end{aligned}$$

Step 3): Find area of $\triangle PCD$.

$$\begin{aligned} & \sin(60^\circ) = \frac{8}{AC} \\ & AC = 9.24 \text{ m} \\ & PC = 9.24 - 3 = 6.24 \text{ m} \\ & \text{Area}_{\triangle PCD} = \frac{1}{2} \times 6.24 \times 3 = 9.36 \text{ m}^2 \end{aligned}$$

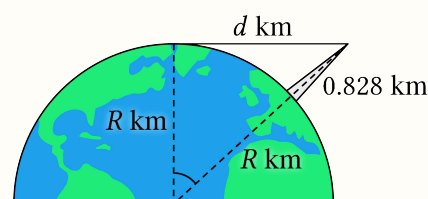
Step 4): Sum up areas.

$$\begin{aligned} \text{Area}_{ABCD} &= \text{Area}_{\triangle APD} + \text{Area}_{\triangle ABC} + \text{Area}_{\triangle PCD} \\ &= 4.5 + 18.48 + 9.36 \\ &= 32.34 \text{ m}^2 \end{aligned}$$

Example 3:

Siddarth wants to estimate the radius of the Earth. He climbs to the top of Burj Khalifa, $h = 828$ m tall, and looks to the horizon d km away from him.

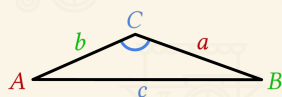
- a) Derive an equation for R in terms of d and h .
b) If he estimates d to be 100 km, estimate R .
c) Calculate the angle subtended at the Earth's center between the horizon and Siddarth.



$$\begin{aligned} \text{a) } & (R + h)^2 = R^2 + d^2 \\ & R^2 + 2hR + h^2 = R^2 + d^2 \\ & 2hR = d^2 - h^2 \\ & R = \frac{d^2 - h^2}{2h} \\ \text{b) } & R = \frac{100^2 - 0.828^2}{2 \times 0.828} = 6038 \text{ km} \\ \text{c) } & \sin(\theta) = \frac{100}{6038 + 0.828} \\ & \theta = 0.95^\circ \end{aligned}$$

Non-right Triangles

Trigonometry isn't just for right-angled triangles, these useful results apply to any triangle.



Area of Triangle

$$\text{Area} = \frac{1}{2}ab \sin(C)$$

where a , b , and c are the side lengths of any triangle $\triangle ABC$, opposite the interior angles at vertices A , B , and C , respectively.

Sine Rule

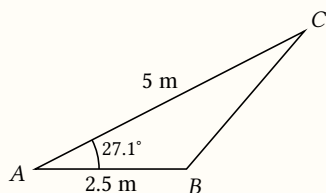
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

Example 1:

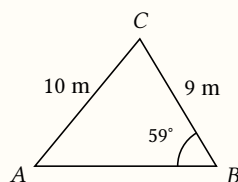
In $\triangle ABC$, $AB = 2.5$ m, $CA = 5$ m, and $\angle CAB = 27.1^\circ$. Find its area.



$$\begin{aligned} \text{Area}_{\triangle ABC} &= \frac{1}{2}ab \sin(C) \\ &= \frac{1}{2} \times 5 \times 2.5 \times \sin(27.1^\circ) \\ &= 2.85 \text{ m}^2 \end{aligned}$$

Example 2:

In $\triangle ABC$, $BC = 9$ m, $CA = 10$ m, and $\angle ABC = 59^\circ$. Find its area.



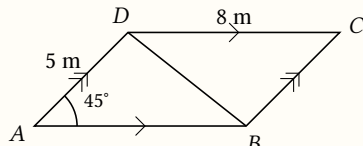
$$\begin{aligned} \frac{a}{\sin(A)} &= \frac{b}{\sin(B)} \\ \frac{10}{\sin(59^\circ)} &= \frac{9}{\sin(\angle CAB)} \\ \angle CAB &= \sin^{-1}\left[\frac{9}{10} \sin(59^\circ)\right] = 50.5^\circ \\ \angle BCA &= 180^\circ - 59^\circ - 50.5^\circ = 70.5^\circ \\ \text{Area}_{\triangle ABC} &= \frac{1}{2} \times 10 \times 9 \times \sin(70.5^\circ) = 42.4 \text{ m}^2 \end{aligned}$$

Example 3:

In the parallelogram $ABCD$, $CD = 8$ m, $DA = 5$ m, and $\angle DAB = 45^\circ$.

a) Find length BD .

b) Find the area of parallelogram $ABCD$.



$$\begin{aligned} \text{a) } AB &= CD = 8 \text{ m (opp. sides are congruent)} \\ BD^2 &= a^2 + b^2 - 2ab \cos(C) \\ BD^2 &= 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos(45^\circ) \\ BD &= 5.69 \text{ m} \\ \text{b) } \text{Area}_{\triangle ABD} &= \frac{1}{2} \times 5 \times 8 \times \sin(45^\circ) = 14.14 \text{ m}^2 \\ \text{Area}_{ABCD} &= 2 \times \text{Area}_{\triangle ABD} = 28.28 \text{ m}^2 \end{aligned}$$

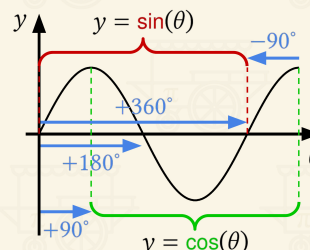
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Wave Trigonometry

While the sine and cosine functions help us understand triangles, they are really wave patterns at heart.

Sine Wave Recap

Let's quickly recap the translational and symmetric properties of the sine wave.

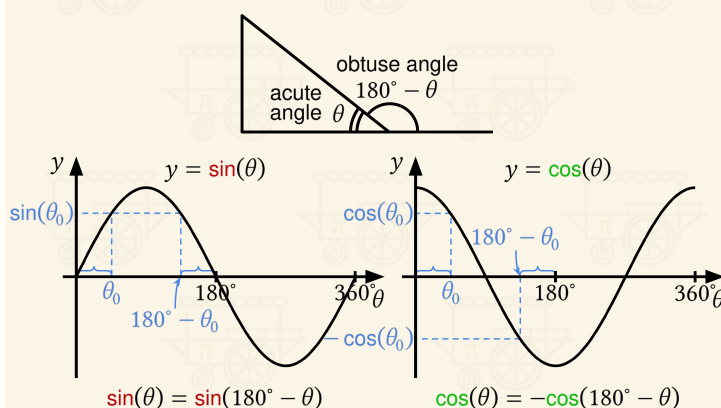


Phase Shift Identities

- $\cos(\theta) = \sin(\theta + 90^\circ)$
- $\sin(\theta) = \cos(\theta - 90^\circ)$
- $\sin(\theta) = -\sin(\theta + 180^\circ)$
- $\cos(\theta) = -\cos(\theta + 180^\circ)$
- $\sin(\theta) = \sin(\theta + 360^\circ)$
- $\cos(\theta) = \cos(\theta + 360^\circ)$

Obtuse Angles

An obtuse angle measures more than 90° but less than 180° . Let's explore how trigonometric functions behave in this range and the connection to previous results.

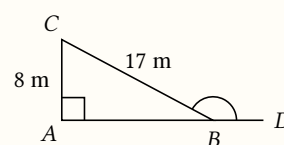


Example 1:

In the figure below, ABD is a straight line, $BC = 15$ m, $CA = 8$ m, $\angle CAB = 90^\circ$.

a) Find $\sin \angle CBD$.

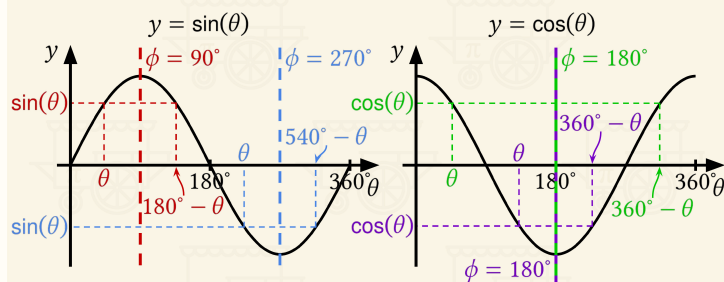
b) Find $\cos \angle CBD$.



$$\begin{aligned} \text{a) } \sin(\angle CBD) &= \sin(180^\circ - \angle CBD) \\ &= \sin(\angle ABC) \\ &= \frac{8}{17} \\ \text{b) } AB &= \sqrt{17^2 - 8^2} = 15 \text{ m} \\ \cos(\angle CBD) &= -\cos(180^\circ - \angle CBD) \\ &= -\cos(\angle ABC) \\ &= -\frac{15}{17} \end{aligned}$$

Multiple Solutions

The periodic and symmetric nature of the sine function allows a single y -value to correspond to multiple θ -values within a given range. In one rotation ($0 \leq \theta \leq 360^\circ$), the two solutions are symmetrically reflected across a line of symmetry (ϕ).



Dual Solution

$$\bullet \theta_2 = 2\phi - \theta_1$$

- $\sin(\theta) > 0 \Rightarrow \theta_2 = 2 \times 90^\circ - \theta_1$
- $\sin(\theta) < 0 \Rightarrow \theta_2 = 2 \times 270^\circ - \theta_1$
- $\cos(\theta) > 0 \Rightarrow \theta_2 = 2 \times 180^\circ - \theta_1$
- $\cos(\theta) < 0 \Rightarrow \theta_2 = 2 \times 180^\circ - \theta_1$

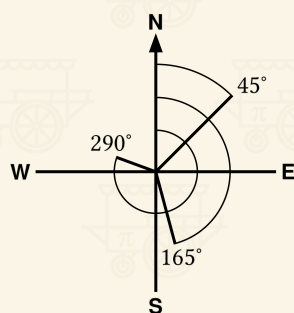
Example 1:

Solve for $0^\circ < \theta < 180^\circ$ in each of the following equations.

- a) $\sin(\theta - 25^\circ) = 0.6$ b) $\cos(\theta + 5^\circ) = -0.4$
 - c) $\cos(2\theta + 15^\circ) = 0.3$
- a) $\sin(\theta - 25^\circ) = 0.6$
 $\theta - 25^\circ = 36.87^\circ$ or $\theta - 25^\circ = 180^\circ - 36.87^\circ$
 $\theta = 61.87^\circ$ or 168.13°
 - b) $\cos(\theta + 5^\circ) = -0.4$
 $\theta + 5^\circ = 113.58^\circ$ or $\theta + 5^\circ = 360^\circ - 113.58^\circ$
 $\theta = 108.58^\circ$ or $\theta = 241.42^\circ$ (rejected, $241.42^\circ > 180^\circ$)
 - c) $\cos(2\theta + 15^\circ) = 0.3$
 $2\theta + 15^\circ = 72.54^\circ$ or $2\theta + 15^\circ = 360^\circ - 72.54^\circ$
 $\theta = 28.77^\circ$ or $\theta = 136.23^\circ$

Bearings

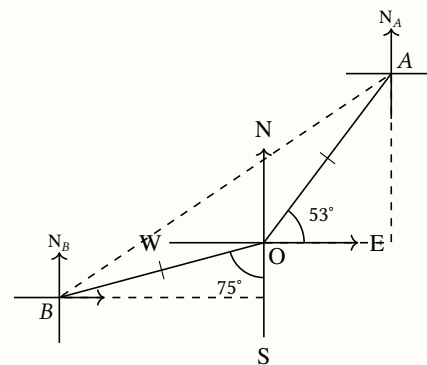
Bearings provide a precise way to describe direction, measured clockwise from north. Trigonometry can be used to solve problems involving angles, distances, and navigation.



Example 1:

The figure shows the position of O , A , and B . Find the bearings of

- a) A from O , b) B from O ,
- c) O from A , d) O from B .
- e) Given that A and B are equidistant from O , find the bearing of B from A .

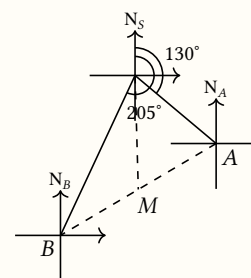


- a) $\text{Bearing}_{AO} = 90^\circ - 53^\circ = 37^\circ$
- b) $\text{Bearing}_{BO} = 180^\circ + 75^\circ = 255^\circ$
- c) $\text{Bearing}_{OA} = 180^\circ + (90^\circ - 53^\circ) = 217^\circ$
- d) $\text{Bearing}_{OB} = 90^\circ - (90^\circ - 75^\circ) = 75^\circ$
- e) $\angle BOA = \angle BON + \angle NOA$
 $= (180^\circ - 75^\circ) + 37^\circ$
 $= 142^\circ$
 $\angle OAB = \frac{180^\circ - 142^\circ}{2}$ (isos. \triangle)
 $= 19^\circ$
 $\text{Bearing}_{BA} = \text{Bearing}_{AO} + \angle OAB$
 $= 217^\circ + 19^\circ$
 $= 236^\circ$

Example 2:

A coast guard station S detects two distress signals from separate boats, A and B . Boat A is located 15 km from the station on a bearing of 130° . Boat B is located 25 km from the station on a bearing of 205° .

- a) Determine the distance between the two boats.
- b) Find the bearing of Boat B from Boat A .
 A rescue helicopter is dispatched from the station, flying at 180 km/h. The helicopter will drop a survival kit at a point M , which is midway between Boat A and Boat B .
- c) Determine the time the helicopter takes to reach point M .



- a) $\angle ASB = 205^\circ - 130^\circ = 75^\circ$
 $AB^2 = 15^2 + 25^2 - 2 \times 15 \times 25 \times \cos 75^\circ$
 $AB = 25.6 \text{ km}$
- b) $\frac{25.6}{\sin(75^\circ)} = \frac{25}{\sin(\angle BAS)}$
 $\angle BAS = 70.5^\circ$
 $\text{Bearing}_{BA} = 360^\circ - (90^\circ - 40^\circ) - 70.5^\circ = 239.5^\circ$
- c) $SM^2 = \left(\frac{25.6}{2}\right)^2 + 15^2 - 2 \times \frac{25.6}{2} \times 15 \times \cos 70.5^\circ$
 $SM = 16.2 \text{ km}$
 $\text{Time Taken} = \frac{16.2}{180}$
 $= 0.08976 \text{ hours}$
 $= 5 \text{ minutes } 23 \text{ seconds}$

3

Triangle Geometry

That triangle kinda looks like this one... Prove it!

Basic Geometric Properties Recap

Here's a summary of basic geometric properties that we'd previously learned.

Lines

Supplementary angles

$$\angle a + \angle b = 180^\circ \quad (\text{supp. } \angle\text{s})$$

$$\angle c + \angle d = 180^\circ \quad (\text{supp. } \angle\text{s})$$

Vertically Opposite angles

$$\angle a = \angle c \quad (\text{vert. opp. } \angle\text{s})$$

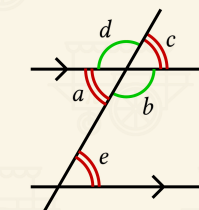
$$\angle b = \angle d \quad (\text{vert. opp. } \angle\text{s})$$

Corresponding angles

$$\angle c = \angle e \quad (\text{corr. } \angle\text{s})$$

Alternate angles

$$\angle a = \angle e \quad (\text{alt. } \angle\text{s})$$



Interior angles

$$\angle b + \angle e = 180^\circ \quad (\text{int. } \angle\text{s})$$

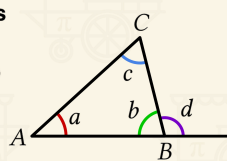
Triangles

Angle sum of triangle

$$\angle a + \angle b + \angle c = 180^\circ \quad (\angle \text{sum of } \triangle)$$

Exterior angle of triangle

$$\angle a + \angle c = \angle d \quad (\text{ext. sum of } \triangle)$$

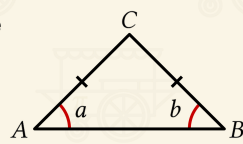


Base angles of isosceles triangle

$$\angle a = \angle b \quad (\text{base } \angle\text{s of isos. } \triangle)$$

Sides of isosceles triangle

$$BC = CA \quad (\text{sides of isos. } \triangle)$$

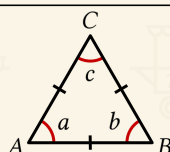


Angles of equilateral triangle

$$\angle a = \angle b = \angle c = 60^\circ \quad (\angle\text{s of equi. } \triangle)$$

Sides of equilateral triangle

$$AB = BC = CA \quad (\text{sides of equi. } \triangle)$$



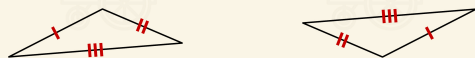
Congruence

Two shapes are congruent if they have the same size and shape, regardless of orientation or position. These tests determine congruence using different sets of information.

Two triangles are congruent if...

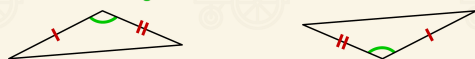
SSS (Side-Side-Side):

...all **three sides**...



SAS (Side-Angle-Side):

...**two sides** and the **angle** between them...



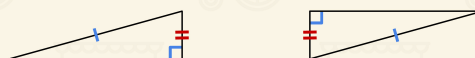
AAS (Angle-Angle-Side):

...**two angles** and **any side**...



RHS (Right-Hypotenuse-Side):

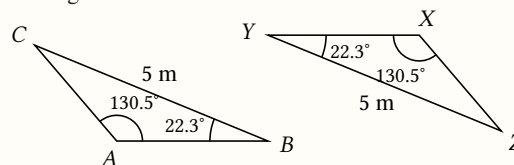
...they are **right-angled**, and the **hypotenuse** and **one other side**...



...in one triangle are **equal** to the corresponding parts in the other triangle.

Example 1:

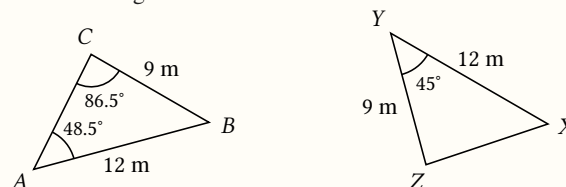
In $\triangle ABC$, $BC = 5$ m, $\angle ABC = 22.3^\circ$, and $\angle CAB = 130.5^\circ$. In $\triangle XYZ$, $YZ = 5$ m, $\angle XYZ = 22.3^\circ$, and $\angle ZXY = 130.5^\circ$. Prove that $\triangle ABC$ and $\triangle XYZ$ are congruent.



- $BC = YZ = 5$ m
- $\angle ABC = \angle XYZ = 22.3^\circ$
- $\angle CAB = \angle ZXY = 130.5^\circ$
- $\therefore \triangle ABC \cong \triangle XYZ$ (AAS congruence test)

Example 2:

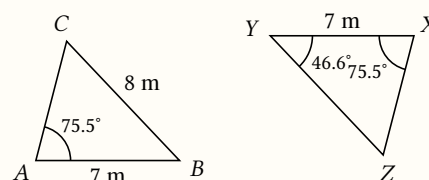
In $\triangle ABC$, $AB = 12$ m, $BC = 9$ m, $\angle BCA = 86.5^\circ$, and $\angle CAB = 48.5^\circ$. In $\triangle XYZ$, $XY = 12$ m, $YZ = 9$ m, and $\angle XYZ = 45^\circ$. Prove that $\triangle ABC$ and $\triangle XYZ$ are congruent.



- $AB = XY = 12$ m
- $BC = YZ = 9$ m
- $\angle ABC = 180^\circ - 86.5^\circ - 48.5^\circ = 45^\circ$
- $\angle ABC = \angle XYZ = 45^\circ$
- $\therefore \triangle ABC \cong \triangle XYZ$ (SAS congruence test)

Example 3:

In $\triangle ABC$, $AB = 7$ m, $BC = 8$ m, and $\angle CAB = 75.5^\circ$. In $\triangle XYZ$, $XY = 7$ m, $\angle XYZ = 46.6^\circ$, and $\angle ZXY = 75.5^\circ$. Using the SSS congruence test, prove that $\triangle ABC$ and $\triangle XYZ$ are congruent.



Step 1): Find the side lengths of $\triangle ABC$.

$$\begin{aligned} \frac{8}{\sin(75.5^\circ)} &= \frac{7}{\sin(\angle BCA)} \\ \angle BCA &= \sin^{-1}\left[\frac{7}{8} \sin(75.5^\circ)\right] = 57.9^\circ \\ \angle ABC &= 180^\circ - 75.5^\circ - 57.9^\circ = 46.6^\circ \\ CA^2 &= 7^2 + 8^2 - 2 \times 7 \times 8 \times \sin(46.6^\circ) \\ CA &= 6 \text{ m} \end{aligned}$$

Step 2): Find the side lengths of $\triangle XYZ$.

$$\begin{aligned} \angle YZX &= 180^\circ - 75.5^\circ - 46.6^\circ = 57.9^\circ \\ \frac{7}{\sin(57.9^\circ)} &= \frac{YZ}{\sin(75.5^\circ)} = \frac{ZX}{\sin(46.6^\circ)} \\ YZ &= \frac{7 \sin(75.5^\circ)}{\sin(57.9^\circ)} = 8 \text{ m} \\ ZX &= \frac{7 \sin(46.6^\circ)}{\sin(57.9^\circ)} = 6 \text{ m} \end{aligned}$$

Step 3): Compare respective sides.

- $AB = XY = 7$ m
- $BC = YZ = 8$ m
- $CA = ZX = 6$ m
- $\therefore \triangle ABC \cong \triangle XYZ$ (SSS congruence test)

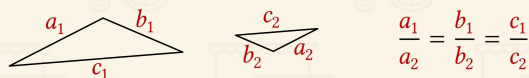
Similarity

Two shapes are similar if they have the same shape but may differ in size. Again, there are tests to determine similarity using different sets of information.

Two triangles are similar if...

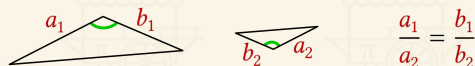
SSS (Side-Side-Side):

...all **three** pairs of corresponding **sides** are in the same **proportion**.



SAS (Side-Angle-Side):

...**one** pair of corresponding **angles** is **equal** and the **sides** forming those angles are in the same **proportion**.



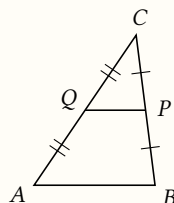
AA (Angle-Angle):

...**two** corresponding **angles** are **equal**.



Example 1:

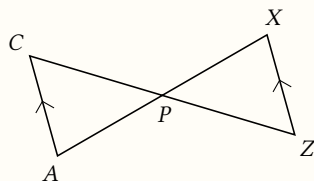
In $\triangle ABC$, $BP = PC$, $CQ = QA$. Prove that $\triangle ABC$ is similar to $\triangle QPC$.



- $\angle BCA = \angle PCQ$ (common angle)
- $CA = 2CQ$
 $BC = 2PC$
 $\frac{PC}{BC} = \frac{CQ}{CA} = \frac{1}{2}$
- $\therefore \triangle ABC \sim \triangle QPC$ (SAS similarity test)

Example 2:

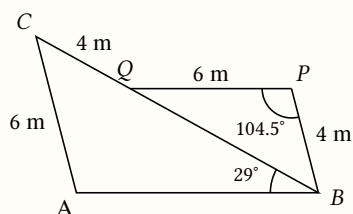
In the figure below, AB is parallel to XY . Prove that $\triangle APB$ is similar to $\triangle XYP$.



- $\angle APB = \angle YPX$ (vert. opp. \angle s)
- $\angle PBA = \angle PXY$ (alt. \angle s)
- $\therefore \triangle APB \sim \triangle XYP$ (AA similarity test)

Example 3:

In $\triangle ABC$, $QC = 4$ m, $CA = 6$ m, and $\angle ABC = 29^\circ$. In $\triangle BPQ$, $BP = 4$ m, $PQ = 6$ m, and $\angle BPQ = 104.5^\circ$. Prove that $\triangle ABC$ is similar to $\triangle BPQ$.



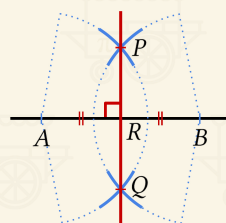
- $QB^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \times \cos(104.5^\circ)$
 $QB = 8$ m

$$\begin{aligned} \frac{6}{\sin(29^\circ)} &= \frac{8+4}{\sin(\angle CAB)} \\ \sin(\angle CAB) &= 0.968 \\ \sin(180^\circ - \angle CAB) &= 0.968 \quad (\text{obtuse } \angle) \\ \angle CAB &= 104.5^\circ \\ \angle BCA &= 180^\circ - 29^\circ - 104.5^\circ = 46.5^\circ \\ AB^2 &= 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos(46.5^\circ) \\ AB &= 9 \text{ m} \\ \frac{AB}{PQ} &= \frac{BC}{QB} = \frac{CA}{BP} \\ \frac{9}{6} &= \frac{12}{8} = \frac{6}{4} = \frac{3}{2} \\ \therefore \triangle ABC &\sim \triangle BPQ \quad (\text{SSS similarity test}) \end{aligned}$$

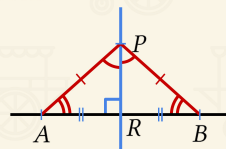
Bisectors

Bisectors are lines that divide angles or line segments into two equal parts. They provide a foundation for understanding key properties of triangles and circles.

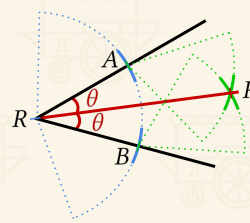
Perpendicular Bisectors



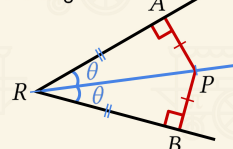
Any point P on the bisector is **equidistant** from the two points it bisects.



Angle Bisectors

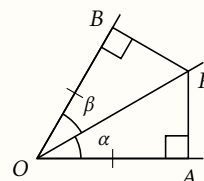


Any point P on the bisector is **equidistant** from the two sides of the angle.



Example 1:

In the figure below, $OA = OB$ and $\angle OAP = \angle PBO = 90^\circ$. Prove that OP is an angle bisector of $\angle BOA$.



- OP is a common side of $\triangle OAP$ and $\triangle OPB$.
- $\Rightarrow \frac{OA}{\cos(\alpha)} = \frac{OB}{\cos(\beta)}$
- $OA = OB$
- $\Rightarrow \frac{OA}{\cos(\alpha)} = \frac{OA}{\cos(\beta)}$
- $\Rightarrow \cos(\alpha) = \cos(\beta)$
- $\Rightarrow \alpha = \beta$ (acute \angle)
- $\therefore OP$ is an angle bisector of $\angle BOA$.

Example 2:

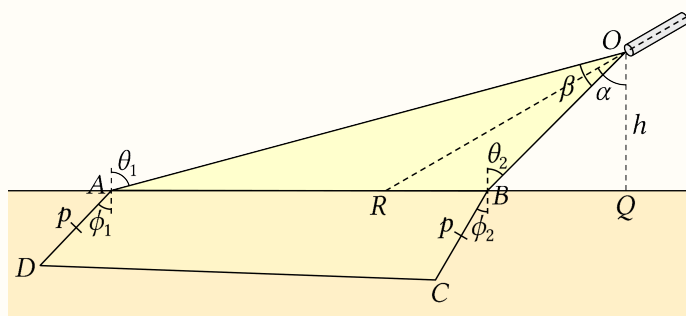
Scientist Alex directs light through an optical fiber onto a patient's skin. The fiber's end is positioned $h = 1$ mm above the skin and is inclined at $\alpha = 60^\circ$. Assume that the light exits the fiber at a point with a divergence angle of $\beta = 30^\circ$. In the figure below, OR bisects $\angle AOB$.

- Calculate the illumination width AB on the skin.
 - Determine the angles of incidence θ_1 and θ_2 for the upper and lower divergence rays on the skin, respectively.
- Given that the relationship between the angle of incidence θ and the angle of refraction ϕ is

$$\sin \theta = 1.4 \sin \phi$$

and that the penetration length of light in skin is $p = 0.75$ mm.

- Find the angles of refraction ϕ_1 and ϕ_2 .
- Hence, determine the area of the quadrilateral $ABCD$.



- $$\tan\left(60 - \frac{30}{2}\right) = \frac{BQ}{1}$$

$$BQ = 1 \text{ mm}$$

$$\tan\left(60 + \frac{30}{2}\right) = \frac{AQ}{1}$$

$$AQ = 3.73 \text{ mm}$$

$$AB = 3.73 - 1 = 2.73 \text{ mm}$$
- $$\theta_1 = \angle AOQ = 75^\circ \quad (\text{alt. } \angle s)$$

$$\theta_2 = \angle BOQ = 45^\circ \quad (\text{alt. } \angle s)$$
- $$\sin(75^\circ) = 1.4 \sin(\phi_1)$$

$$\phi_1 = 43.6^\circ$$

$$\sin(45^\circ) = 1.4 \sin(\phi_2)$$

$$\phi_2 = 30.3^\circ$$
- $$\angle DAB = 43.6^\circ + 90^\circ = 133.6^\circ$$

$$BD^2 = 0.75^2 + 2.73^2 - 2 \times 0.75 \times 2.73 \times \cos(133.6^\circ)$$

$$BD = 3.29 \text{ mm}$$

$$\angle ABC = 90 - 30.3^\circ = 59.7^\circ$$

$$\text{Area}_{ABCD} = \text{Area}_{\triangle ABD} + \text{Area}_{\triangle BCD}$$

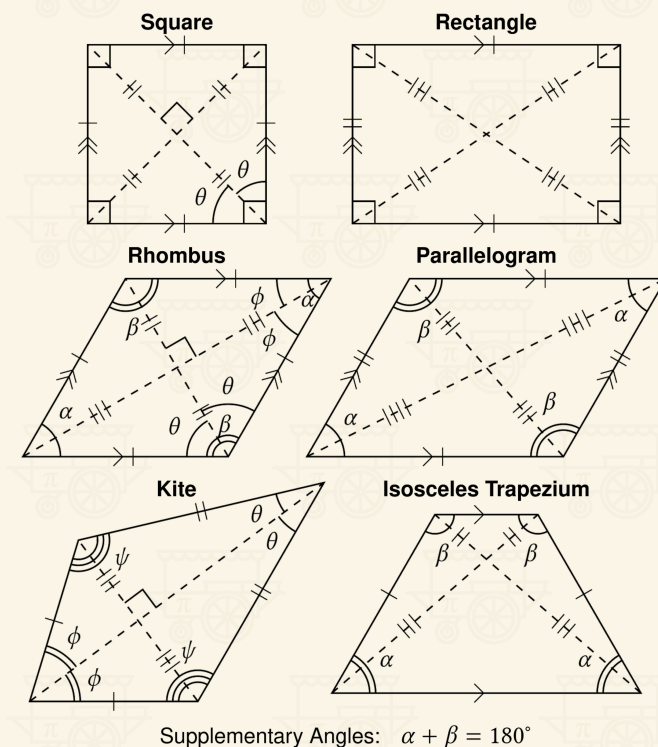
$$= \frac{1}{2} \times 0.75 \times 2.73 \times \sin(133.6^\circ)$$

$$+ \frac{1}{2} \times 3.29 \times 0.75 \times \sin(59.7^\circ)$$

$$= 1.81 \text{ mm}^2$$

Quadrilateral Properties Recap

Not quite triangles, but quadrilaterals are closely related to them.


Glossary

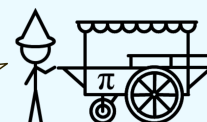
Here's a short description of new symbols we've encountered.

- $\triangle ABC$: a triangle with vertices A , B , and C .
- $\angle ABC$: an angle between lines AB and BC .
- $A \perp B$: means " A is perpendicular to B ".
- $A \parallel B$: means " A is parallel to B ". Some textbooks use $A // B$, but that is not standard notation.
- $A \cong B$: means " A is congruent to B ".
- $A \sim B$: means " A is similar to B ".
- \therefore : means "therefore", usually used for conclusions.
- \rightarrow : parallel marks represent parallel lines.
- $+$: tick marks represent lines of equal length.

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See you in
Geometry part 2!



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